Trigonometric Identities - Math 142

(*You can use this handout on tests and quizzes.*)

Pythagorean identities:

$$\sin^2\alpha + \cos^2\alpha = 1, \quad 1 + \cot^2\alpha = \csc^2\alpha, \quad \tan^2\alpha + 1 = \sec^2\alpha$$

Sum and Difference Identities:

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta, \quad \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta, \quad \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}, \quad \tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Product-to-sum Identities:

$$\sin A\cos B = \tfrac{1}{2}(\sin(A+B)+\sin(A-B)), \quad \sin A\sin B = \tfrac{1}{2}(\cos(A-B)-\cos(A+B))$$
$$\cos A\sin B = \tfrac{1}{2}(\sin(A+B)-\sin(A-B)), \quad \cos A\cos B = \tfrac{1}{2}(\cos(A-B)+\cos(A+B))$$
$$\sin^2 A = \frac{1-\cos(2A)}{2}, \quad \cos^2 A = \frac{1+\cos(2A)}{2}$$

Sum-to-product Identities:

$$\sin x + \sin y = 2\sin\left(\tfrac{x+y}{2}\right)\cos\left(\tfrac{x-y}{2}\right), \quad \cos x + \cos y = 2\cos\left(\tfrac{x+y}{2}\right)\cos\left(\tfrac{x-y}{2}\right)$$
$$\sin x - \sin y = 2\cos\left(\tfrac{x+y}{2}\right)\sin\left(\tfrac{x-y}{2}\right), \quad \cos x - \cos y = -2\sin\left(\tfrac{x+y}{2}\right)\sin\left(\tfrac{x-y}{2}\right)$$

Derivative Identities:

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}} \quad \text{when } |x|<1$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}, \quad \frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2} \quad \text{when } x\in\mathbb{R}$$

$$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}, \quad \frac{d}{dx}(\csc^{-1}x) = -\frac{1}{|x|\sqrt{x^2-1}} \quad \text{when } |x|>1$$

Integral Identities:

$$\int \sec^2 x dx = \tan x + C, \quad \int \csc^2 x dx = -\cot x + C$$

$$\int \sec x\tan x dx = \sec x + C, \quad \int \csc x\cot x dx = -\csc x + C$$

$$\int \tan x dx = \ln|\sec x| + C, \quad \int \cot x dx = \ln|\sin x| + C$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C, \quad \int \csc x dx = -\ln|\csc x + \cot x| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}(\tfrac{x}{a}) + C, \quad \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}(\tfrac{x}{a}) + C$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}|\tfrac{x}{a}| + C$$